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The flow of granular solids through circular orifices

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It has been shown that the use of the bulk density term in place of the particle density, in the equation of flow for granular solids passing through a circular orifice, very largely eliminates differences due to the shape, rugosity, density, porosity and friction of the particles.

The equation

$$\mathrm{Do} = (1{\cdot}136 + 0{\cdot}000173\mathrm{Dp})\left(\frac{4\mathrm{W}}{60\pi\rho\mathrm{B}\sqrt{g}}\right)^{\frac{1}{0{\cdot}903+0{\cdot}675\log \mathrm{Dp}}}$$

has been tested on seven different materials and has been found to predict the flow of single and binary systems with an overall accuracy of ±5% and ±10% respectively.

The flow of granular materials through circular orifices has been studied previously and equations have been derived which allow predictions of the flow rate to be made. Most of these equations have been either empirical or based on dimensional analysis (Deming & Mehring, 1929; Bingham & Wikoff, 1931; Rose & Tanaka, 1959; Fowler & Glastonbury, 1959; Brown & Richards, 1959). Relatively few have been based on theoretical considerations (Brown, 1961; Zenz, 1962; McDougall & Evans, 1965; Shinohara, Demitsu & others, 1968).

Brown & Richards (1959) proposed a dimensionally balanced equation* of the form

$$(\mathrm{Do} - \mathrm{k}) = \mathrm{A}\left(\frac{4\mathrm{W}}{60\pi\rho\mathrm{p}\sqrt{g}}\right)^{0{\cdot}4} \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

where k was a measure of the width of the empty annulus observed at the periphery of the orifice. This equation was found to apply to materials such as coal, glass beads, tapioca and sand flowing through a wide range of orifice sizes (Brown & Richards, 1959). Variation in the parameters A and k were shown to be due, *inter alia*, to variations in particle size and shape of the materials.

Jones and Pilpel (1966) considered one material, magnesia, which was available in a large range of particle sizes, and were able to study the effect of particle size without the complicating effects of variation in shape, rugosity and density, etc.

By writing equation (1) in the form

$$\mathrm{Do} = \mathrm{A}\left(\frac{4\mathrm{W}}{60\pi\rho\mathrm{p}\sqrt{g}}\right)^{\frac{1}{\mathrm{n}}} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$

they were able to show that the parameters A and n were functions of particle size. Although difficulties then arise in regard to the dimensions of the terms A and n, which in equation (1) were dimensionless, having established a numerical relation between A and n and particle size, predictions could then be made of the flow rate of any other given size fraction.

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* For rotation see p. 729.

For equation (2) to have practical importance it would be necessary to include a term or terms which would account for variations in shape and rugosity of the particles of different materials. Previous equations have involved angular properties (Takahashi, 1935; Franklin & Johanson, 1955), others have included a shape factor (Rose & Tanaka, 1959; Ahmad & Pilpel, 1969). Many workers have used a bulk density term (Fowler & Glastonbury, 1959; McDougall & Evans, 1965; Beverloo, Leniger & Van der Velde, 1961) since this embodies the shape, rugosity and frictional characteristics of the materials.

The high correlation of bulk density and flow rate has been pointed out in a recent paper (Sumner, Thompson & others, 1966) and Delaplaine (1956) has shown that the bulk density of a flowing bed is only 0·02 units lower than that of the static bed.

An advantage of bulk density is that it compensates for differences between the apparent and effective particle densities: differences which may be very large (up to 40%) in the case of granulated cohesive materials (Harwood & Pilpel, 1968).

This study is a test of the use of the bulk density term instead of the particle density term to establish an equation of the same form as equation (2), which can be applied to materials that differ considerably in the shape, rugosity, density and frictional characteristics of their particles.

EXPERIMENTAL

Materials

The materials tested were smooth and irregular griseofulvin granules, silica sand and glass beads. The reported results for magnesia (Jones & Pilpel, 1966) and for smooth and irregular lactose granules (Ahmad & Pilpel, 1969), obtained using the same apparatus, have been included to extend the generality of the results obtained.

The materials were separated into sieve fractions on British Standard sieves and surface fines were removed by sieving 20–40 g portions on an Alpine Airjet sieve for 3 min. The samples were dried in an air oven and stored in stoppered glass jars.

The tap and bulk densities were measured using a standard apparatus (British Standard, 1948). The particle densities were measured using the specific gravity bottle method.

Some of the physical properties of the materials are given in Table 1.

Table 1. *Sieve fractions and densities of granular materials*

Material	B.S.S. size	Arithmetic mean size (μm)	Density (g cm^{-3}) Particle	Bulk	Tap
Irregular griseofulvin granules	60–85	215	1·430	0·507	0·551
	44–60	300	1·431	0·463	0·500
	25–44	430	1·433	0·407	0·456
	22–25	655	1·428	0·393	0·438
	16–22	855	1·435	0·385	0·435
	10–16	1340	1·435	0·378	0·419
	8–10	1866	1·421	0·374	0·419
Smooth griseofulvin granules	16–22	855	1·448	0·556	0·609
	12–16	1200	1·443	0·538	0·579
	10–12	1540	1·432	0·522	0·560
	8–10	1866	1·421	0·507	0·556
	6–8	2435	1·403	0·500	0·551

Table 1—*continued*

Material	B.S.S. size	Arithmetic mean size (μm)	Density (g cm^{-3})		
			Particle	Bulk	Tap
Sand	<85	<180	2·653	1·212	1·437
	60–85	215	2·625	1·317	1·543
	44–60	300	2·642	1·371	1·594
	25–44	475	2·698	1·434	1·661
	16–25	800	2·823	1·464	1·661
Glass beads	—	<53	2·965	1·289	1·409
	—	113	2·962	1·716	1·876
	—	150	2·970	1·737	1·765
	60–85	213	2·973	1·746	1·886
	36–52	368	2·978	1·818	1·848
	25–44	486	2·981	1·717	1·855
	—	605	2·979	1·774	1·896
Magnesia	300–350	48	3·439	1·000	1·095
	150–200	90	3·458	0·920	0·988
	72–150	158	3·431	0·903	0·985
	36–52	358	3·456	0·856	0·938
	22–36	561	3·458	0·870	0·930
	16–22	851	3·445	0·860	0·930
	10–16	1340	3·460	0·860	0·941
	8–10	1866	3·456	0·856	0·933
Rounded lactose granules	72–150	160	1·535	0·672	0·738
	52–72	252	1·526	0·556	0·608
	36–52	358	1·535	0·511	0·563
	22–36	560	1·536	0·505	0·542
	16–22	851	1·500	0·495	0·529
	12–16	1201	1·541	0·484	0·529
	10–12	1538	1·535	0·483	0·542
	8–10	1866	1·536	0·501	0·536
	6–8	2435	1·550	0·489	0·535
Irregular lactose granules	72–150	160	1·544	0·555	0·645
	52–72	252	1·551	0·519	0·588
	36–52	358	1·523	0·512	0·575
	22–36	560	1·501	0·488	0·555
	16–22	851	1·563	0·481	0·548
	12–16	1201	1·544	0·475	0·535
	10–12	1538	1·502	0·476	0·525

Apparatus

The apparatus was as used by Jones & Pilpel (1966). It consisted of a vertical copper tube 30 cm long and 3·82 cm internal diameter. A Perspex base plate held a shutter and a sliding orifice plate into which six circular orifices with mean diameters of 0·6–1·7 cm had been cut.

Precautions

In measuring the flow rate from a vertical copper tube, certain restrictions have been well established by previous authors in order to avoid the complicating effects of apparatus geometry (for review see Jones, 1966). End effects arising during the flow measurements are eliminated by measuring the flow rate only when steady conditions are obtained, that is, over the central 3/5ths (approximately) of the flowing column.

Blocking of the orifice will occur when the particle size, Dp, is approximately 1/6th of the orifice size, Do. The conditions must be such that Do $\geq$ 6Dp. Finally, the column diameter Dc must be such that Dc $\geq$ 2·5 Do to eliminate wall effects (Beverloo, Leniger & Van der Velde, 1961; Brown & Richards, 1959; Rose & Tanaka, 1959).

Procedure

With the above reservations in mind the column was filled with the material. The mass emerging from the various sized orifices was then measured in time intervals ranging from 5–60 s. Each measurement was made in triplicate and it was found that the maximum variation between separate determinations was ±5%.

RESULTS

The measured flow rates for the 42 size fractions of the seven different materials when flowing through six orifice sizes are given in Table 2.

The effect of particle size on the flow rate is shown in Fig. 1. The curves follow the anticipated form (Rose & Tanaka, 1959; Jones & Pilpel, 1966) where the flow rate increases with decrease of particle size to a maximum at approximately 200 μm and then falls rapidly as the cohesive forces become increasingly effective.

The present study has been concerned only with free flowing materials and for this reason only those size fractions above 200 μm have been considered in the analysis to follow.

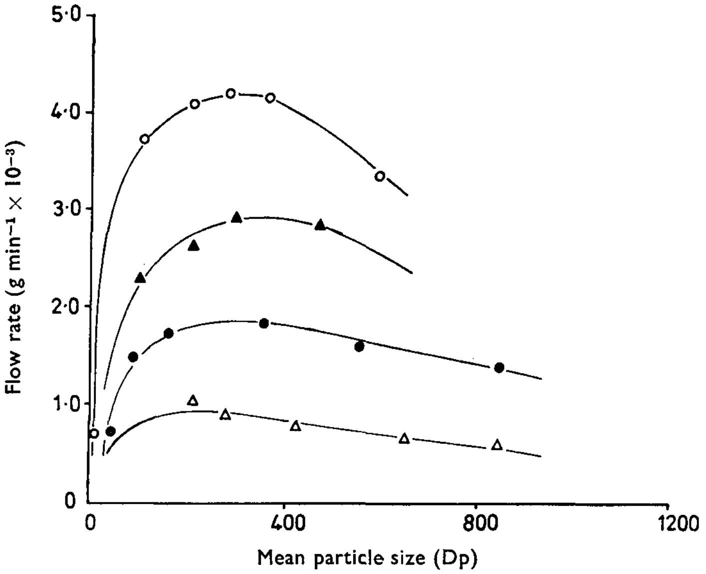

FIG. 1. Flow rate versus particle size for different materials passing through a 1·30 cm orifice. O, Glass beads; ▲, sand; ●, magnesia and △, irregular griseofulvin.

DISCUSSION

From equation (2) it is seen that a plot of the logarithm of Do against the logarithm of $\left(\frac{4W}{60\pi\rho p\sqrt{g}}\right)$ should give a straight line, the slope of which is 1/n and the intercept log A. The values of 1/n and A have been shown to depend on particle size, and, since in the present study 42 size fractions were available and the results were required using both particle density and also the bulk density, the readings were subjected to regressional analysis using a digital computer (Elliott 803) and the correlation coefficients were in all cases better than 0·93. The calculated values of both n and A are given in Table 3.

Table 2. *Flow rates for griseofulvin, sand, glass beads, magnesia and lactose* (g min^{-1})

Mean particle size (μm)	Orifice diameter (cm)					
	0·605	0·707	0·900	1·130	1·330	1·650
Griseofulvin Irregular						
1866	B	B	B	207–221	383–398	836–846
1340	B	82–86	124–125	275–281	456–484	907–964
855	47–49	109–112	160–165	347–349	565–575	1075–1109
655	59–61	129–130	185–188	399–400	659–662	1175–1198
430	75–76	157–159	220–226	462–475	753–768	1411–1441
300	99–100	206–207	283–287	579–584	879–897	B
215	120–121	235–243	302–331	604–618	1044–1082	B
Smooth						
2435	B	B	B	230–254	490–510	950–971
1866	B	B	150–163	352–354	592–605	1105–1132
1540	B	140–142	199–204	446–451	727–746	1310–1380
1200	81–85	180–182	255–260	549–560	923–938	1542–1692
851	105–105	224–229	313–320	651–667	1063–1075	1732–1847
Sand						
800	—	583–614	780–792	1541–1549	2470–2600	4750–4772
475	—	589–611	912–949	1682–1702	2841–2849	5023–5066
300	—	698–726	1045–1075	1722–1744	2911–2949	4595–4696
215	—	624–640	941–959	1573–1617	2637–2661	4680–4692
Glass beads						
605	—	514–545	1256–1284	2320–2340	3657–3735	—
485	—	515–554	1210–1216	2138–2190	3500–3510	—
368	—	715–742	1501–1525	2572–2625	4154–4194	—
284	—	728–762	1526–1546	2709–2729	4203–4217	—
213	—	701–711	1505–1515	2562–2580	4087–4120	—
Mean particle size (μm)	Orifice diameter (cm)					
	0·603	0·740	0·898	1·140	1·353	1·686
Magnesia						
1866	B	B	B	524–560	936–953	1788–1836
1340	B	B	322–388	484–692	1128–1158	2129–2148
851	135–140	241–249	428–453	860–870	1401–1413	2598–2650
560	171–175	288–305	501–538	1015–1020	1610–1630	2798–2860
358	208–212	352–358	610–621	1158–1175	1835–1905	3010–3062
252	245–255	397–404	670–684	1218–1262	1916–1958	3034–3148
160	248–260	398–410	638–678	1180–1210	1760–1784	2684–2840
Mean particle size (μm)	Orifice diameter (cm)					
	0·58	0·75	0·86	1·10	1·31	1·60
Lactose Irregular						
1540	B	B	B	318–331	547–568	1005–1035
1201	B	B	149–164	345–358	590–612	1090–1140
851	B	123–129	188–199	390–408	664–675	1221–1242
560	74–82	154–162	230–246	474–495	806–834	1427–1476
358	95–103	192–216	293–314	550–592	972–1044	—
252	109–118	216–230	318–342	626–648	982–1038	—
160	121–130	226–244	325–347	615–654	1054–1096	—
Smooth						
1540	B	B	156–165	336–358	585–614	1095–1148
1201	B	120–128	180–192	401–408	664–690	1208–1251
851	66–69	140–161	224–236	468–486	780–798	1356–1392
560	78–92	180–192	260–282	548–574	880–912	1500–1564
358	106–114	225–234	320–335	638–652	1000–1022	1622–1672
252	126–130	261–272	368–378	743–756	1190–1215	1756–1804
160	160–170	314–330	456–484	878–892	1404–1436	2092–2140

B denotes orifice blocked.

Table 3. *Values of constants A and n*

Mean particle size Dp (μm)	Log Dp	n	A	
			Using ρB	Using ρp
Griseofulvin Irregular				
1840	3·265	3·447	1·477	2·190
1340	3·127	2·929	1·409	2·224
855	2·932	2·965	1·324	2·152
655	2·816	2·858	1·280	2·016
430	2·633	2·801	1·222	1·919
300	2·477	2·629	1·198	1·840
215	2·332	2·547	1·188	1·784
Smooth				
2435	3·387	3·536	1·509	2·032
1866	3·271	3·291	1·442	2·007
1540	3·188	2·793	1·363	1·961
1200	3·079	2·865	1·274	1·802
851	2·930	2·733	1·222	1·736
Magnesia				
1866	3·271	3·080	1·496	1·782
1340	3·127	2·930	1·416	1·690
851	2·930	2·874	1·305	1·573
560	2·748	2·746	1·251	1·515
358	2·554	2·639	1·186	1·455
252	2·401	2·489	1·174	1·438
160	2·204	2·360	1·214	1·492
Sand				
800	2·903	2·535	1·246	1·248
475	2·677	2·564	1·197	1·208
300	2·477	2·271	1·159	1·195
215	2·332	2·395	1·175	1·329
Glass beads				
605	2·782	2·731	1·175	1·096
485	2·686	2·706	1·189	1·120
368	2·566	2·584	1·129	1·041
284	2·453	2·577	1·119	1·030
213	2·328	2·546	1·116	1·043
Lactose Irregular				
1540	3·188	3·056	1·416	2·046
1201	3·079	3·160	1·377	1·967
851	2·930	2·995	1·332	1·935
560	2·748	2·878	1·257	1·839
358	2·554	2·815	1·194	1·735
252	2·401	2·693	1·180	1·736
160	2·204	2·625	1·198	1·734
Smooth				
1540	3·188	3·032	1·405	1·995
1201	3·079	3·039	1·335	1·923
851	2·930	2·972	1·279	1·844
560	2·748	2·852	1·229	1·785
358	2·554	2·671	1·179	1·750
252	2·401	2·632	1·157	1·673
160	2·204	2·552	1·161	1·576

In Figs 2 and 3 the values of A, using ρp and ρB respectively, are plotted against particle size. It can be seen immediately that by using ρB a good correlation is found between A and the particle size for all of the materials examined. Applying regressional analysis to obtain the best straight line through the points in Fig. 3 gives the relation $A = 1{\cdot}1356 + 0{\cdot}000173\ \text{Dp}$. The correlation coefficient for this line is 0·940 which represents an excellent fit for all the points.

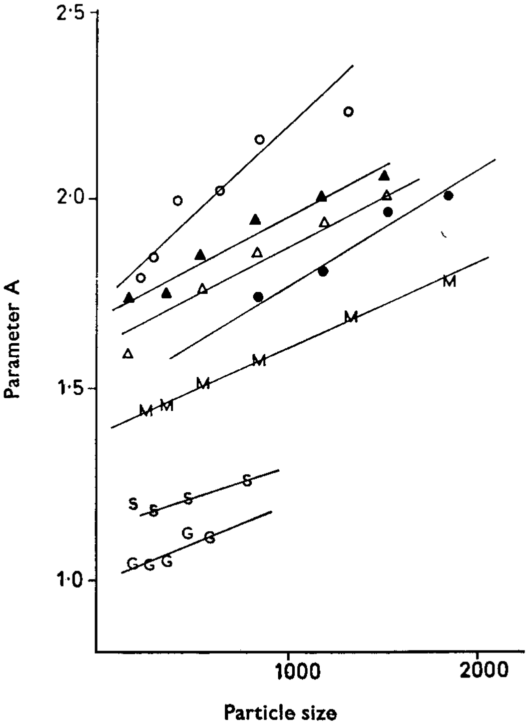

FIG. 2. Variation of parameter A with particle size using ρp. ○—○, Irregular griseofulvin; △—△, irregular lactose; ▲—▲, smooth lactose; ●—●, smooth griseofulvin; M—M, magnesia; S—S, sand and G—G, glass beads.

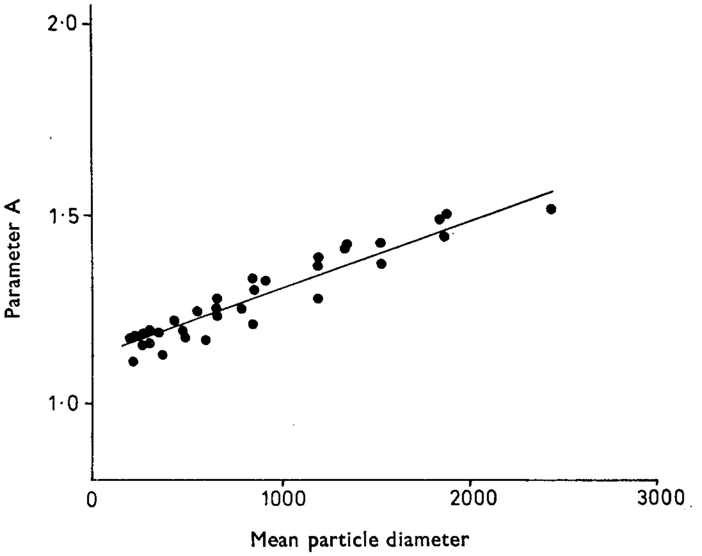

FIG. 3. Variation of parameter A with particle size using ρB (all materials).

Table 4. *Comparison of predicted and measured flow rates*

Material	Orifice size Do (cm)	Mean particle size Dp (μm)	Flow rate (g min^{-1}) Found	Flow rate (g min^{-1}) Calculated	Error %
Griseofulvin					
Irregular	0·707	855	109–112	101	−7
	1·130	300	579–584	595	+2
	1·330	1340	456–484	520	+7
Smooth	0·900	1540	199–204	204	0
	1·650	1200	1542–1692	1480	−4
	1·650	851	1731–1847	1720	−1
Magnesia	0·740	252	397–404	397	0
	1·140	560	1015–1020	1030	+1
	1·686	1866	1788–1836	2030	+11
Sand	0·707	215	624–640	547	−12
	1·140	475	1682–1702	1763	+3
	1·650	800	4750–4772	4620	−3
Glass beads	0·707	284	728–762	689	−5
	1·140	485	2138–2190	2190	+1
	1·330	605	3657–3735	3250	−11
Lactose					
Irregular	0·580	252	109–118	127	+7
	0·860	560	230–246	265	+8
	1·600	1540	1005–1035	1066	+3
Smooth	0·750	252	261–272	259	−1
	1·310	560	880–912	890	0
	1·600	1201	1208–1251	1210	0

Table 5. *Comparison of measured and predicted flow rates for binary mixtures of magnesia*

Dp (μm)	Mixture Concn (% w/w)	Mixture Dp_2 (μm)	Mixture Concn (% w/w)	ρB g/cm³	Do (cm)	W calc. (g/min)	W obs. (g/min)	Error %
253	10	1340	90	0·910	0·898	420	321–357	+17·6
				0·910	1·686	2761	2184–2431	+13·6
	50		50	0·965	0·898	588	489–533	+10·3
				0·965	1·686	3410	698–706	+11·4
	90		10	0·954	0·898	682	698–706	− 2·3
				0·954	1·686	3525	3168–3268	+ 7·9
561	10	1340	90	0·900	0·898	392	328–334	+17·0
				0·900	1·686	2627	2256–2292	+14·6
	50		50	0·888	0·898	458	431–435	+ 8·5
				0·888	1·686	2988	2686–2754	+ 5·3
	90		10	0·875	0·898	522	507–517	+ 5·2
				0·875	1·686	3065	2837–2914	+ 1·0
851	10	561	90	0·877	1·140	1035	1036–1040	0
				0·877	1·686	3095	3040–3092	0
	50		50	0·878	1·140	997	963–988	+ 1·0
				0·878	1·686	3048	2842–2911	+ 4·6
	90		10	0·867	1·140	928	914–921	+ 1·0
				0·867	1·686	2895	2753–2830	+ 2·3
253	20	851	80	0·930	0·603	173	149–151	+14·6
				0·930	1·686	3215	2758–2880	+11·6
	40		60	0·958	0·603	198	176–190	+ 4·2
				0·958	1·686	3398	3074–3160	+ 7·5
	60		40	0·955	0·603	218	213–242	0
				0·955	1·686	3485	3168–3223	+ 8·1

The relation between n and log Dp is shown in Fig. 4 for all of the materials tested. The regression line for all these points was found to be n = 0·9034 + 0·6748 log Dp. The correlation coefficient was 0·844 which signifies a good fit for all the materials.

Thus using the bulk density in equation (2) leads to a general equation of the form

$$\mathrm{Do} = (1{\cdot}136 + 0{\cdot}000173\mathrm{Dp})\left(\frac{4\mathrm{W}}{60\pi\rho\mathrm{B}\sqrt{g}}\right)^{\frac{1}{0{\cdot}903\,+\,0{\cdot}675\,\log \mathrm{Dp}}} \quad \ldots \quad \ldots \quad (3)$$

for relating the flow rate to the orifice size and the particle size of the material.

This equation has been tested by comparing the calculated values with the experimental values of flow rates and the results are shown in Table 4. It can be seen that the average agreement is ±5% which, considering the wide range of materials tested, is regarded as very satisfactory.

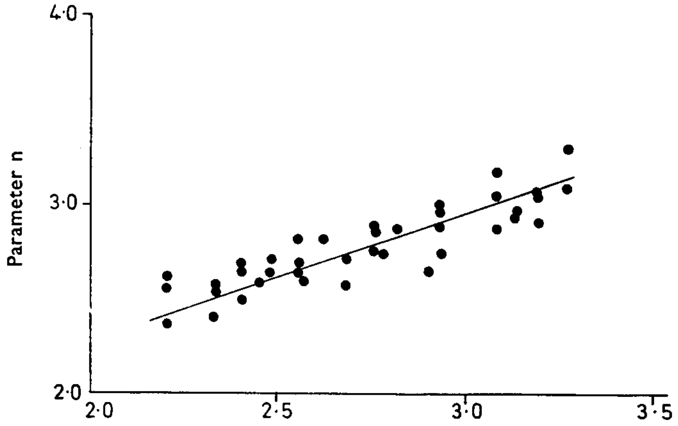

FIG. 4. Variation of n with log Dp.

To further test the validity of equation (3), it has been used to predict the flow rates of some binary mixtures of two different size fractions. These mixtures were prepared by a standard procedure (Jones & Pilpel, 1966), the values of Dp for substituting into equation (3) being taken as geometric means.

Table 5 shows that the agreement between the observed and predicted flow rates for the mixtures was about ±10%, which was again very satisfactory.

The remaining errors are probably due to the use of sieving as a method for classifying and measuring particle size, to segregation of particles in mixtures of sizes and to the use of the B.S. method for measuring bulk density. It is possible that a better method would be to measure the bulk density after fluidizing the sample and then allowing it to settle by slowly reducing the air flow. This value should be closer to that of the flowing material, which was shown by Delaplaine (1956) to be 0·02 units lower than the static bulk density.

In conclusion it should be noted that further work on a variety of materials containing a range of particle sizes will be desirable to establish the generality of the present findings for predicting the flow rates of granular pharmaceuticals.

Notation

A: An empirical function of Dp; Dc: Tube diameter, cm; Do: Orifice diameter, cm; Dp: Particle diameter, μm; g: Acceleration due to gravity, cm s^{-2}; k: A function of the empty annulus dependent on Dp; n: An empirical function of Dp; ρp: Apparent particle density, gcm^{-3}; ρB: Bulk density, g cm^{-3}; W: Flow rate, g min^{-1}.

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